Sheikh 18/10

L-1/T-2/IPE Date: 31/08/2025

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-2 B. Sc. Engineering Examinations 2023-2024

Sub: **MATH 193** (Vector, Matrix and Solid Geometry)

Full Marks: 280 Time: 3 Hours

The figures in the margin indicate full marks.

Symbols have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Prove that the area of the triangle formed by joining mid-point of one of the non-parallel sides of a trapezium to the extremities of the opposite side is half of that of the trapezium. **(16)**

   (b) $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ are the points $\hat{i}-\hat{k}, -\hat{i}+2\hat{j}, 2\hat{i}-3\hat{k}, 3\hat{i}, -2\hat{j}-\hat{k}$ respectively. Find projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ and that of $\overrightarrow{CD}$ on $\overrightarrow{AB}$ and comment on our finding. Also find the cosine of their inclinations. **(10)**

   (c) Find the condition for the equations $\mathbf{p} \times \mathbf{a} = \mathbf{b}$ and $\mathbf{p} \times \mathbf{c} = \mathbf{d}$ to be consistent. Assuming the condition for consistency to be satisfied, solve the equations for the vector $\mathbf{p}$. **($20\frac{2}{3}$)**

2. (a) If **a, b, c** and **d** have a common initial point O and $\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c}=\mathbf{d}$ with $\lambda+\mu+\gamma=1$, then show that the terminal points of **a, b, c** and **d** are coplanar. **(16)**

   (b) Express a vector **p** as a linear combination of vector '**q**' and another vector perpendicular to '**q**' and coplanar with **p** and **q**. **(15)**

   (c) Give the geometrical interpretation of scalar triple product? If **a, b, c,** are three non-coplanar vectors, then express $\boldsymbol{b \times c}$, $\boldsymbol{c \times a}$, $\boldsymbol{a \times b}$ in terms of **a, b, c**. **($15\frac{2}{3}$)**

3. (a) Let A given below be the augmented matrix for a linear system. **($20\frac{2}{3}$)**

$$\begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$$

   Find for what values of 'a' and 'b', the system has

   i) a unique solution.

   ii) a parametric solution.

   iii) no solution.

Contd P/2

30

MATH 193/IPE

(Contd ... Q. No. 3)

(b) Using elementary row transformations find the inverse of the matrix **(16)**

$$A = \begin{bmatrix} 1 & 2 & -2 & -1 \\ -1 & -4 & 4 & 1 \\ 2 & -7 & 4 & -7 \\ 1 & 6 & -5 & 1 \end{bmatrix}$$

(c) If A and B are symmetric matrices, prove that AB is symmetric if and only if A and B commute. **(10)**

4. (a) Reduce the real quadratic form $q = x_1^2 + 2x_2^2 - 2x_3^2 + 4x_1x_2 + 6x_3x_1$ to the canonical form and find the rank, index and signature of the form. **($20\frac{2}{3}$)**

(b) Define eigenvalues and eigenvectors. Let $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$; **(26)**

i) Find the eigenvalues and corresponding eigenvectors.

ii) Is A diagonalizable? Justify your conclusion to find P that diagonalizes the matrix A.

iii) State and verify Cayley-Hamilton theorem for the matrix A, and find A^{-1}.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

Symbols have their usual meaning.

5. (a) Identify the type of the conic section represented by the equation: **($26\frac{2}{3}$)**

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$$

Then, reduce it to its standard form using appropriate rotation and translation of axes, and sketch the curve.

(b) A line makes angles α, β, γ, δ with the four diagonals of a cube, show that **(20)**

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}.$$

6. (a) Find the equation of the plane through the intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$. **(16)**

(b) Find the equation of the plane passing through (2,3,4) and perpendicular to x-axis. Also show that $4x - 2y - 7 = 0$ represents a plane perpendicular to xy-plane. **(10)**

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MATH 193/IPE

(Contd ... Q. No. 6)

(c) ~~Show that~~ Determine whether the four points (2,-1,3), (1,0,1), (0,2,2), and (3,1,0) are coplanar, ~~and~~ If so, find the equation of the common plane. **($20\frac{2}{3}$)**

7. (a) Find an equation for the line that is formed by the intersection of the planes $2x-4y+4z=6$ and $6x+2y-3z=4$. **(16)**

(b) Find the point in which the line $x+y-z-6=0=2x-y+3z+3$ cuts the plane $8x-2y+3z+16=0$. **(10)**

(c) Find the length and the equations of the shortest distance between the lines **($20\frac{2}{3}$)**

$5x-y-z=0=x-2y+z+3$ and $7x-4y-2z=0=x-y+z+3$.

8. (a) The equation $ax^2+4y^2+az^2+6x-8z+d=0$ represents a sphere of radius 3. Find the values of a and d. **(16)**

(b) Find the co-ordinates of the centre and radius of the circle **(15)**

$$x^2+y^2+z^2-2y-4z=11,\ x+2y+2z=15$$

(c) Find the equations of the tangent planes to the surface $\frac{x^2}{4}+y^2-2z^2=1$ which are perpendicular to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-4}$. **($15\frac{2}{3}$)**